1-1-2016

# Risky utilities

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## Abstract

We develop a theory of "risky utilities," i.e., private firms that manage an infrastructure for public service and that may be tempted to engage in excessively risky activities, such as reducing maintenance expenditures (at the risk of provoking a breakdown of the system) or in speculation (at the risk of incurring massive losses it cannot bear). These risky utilities include financial utilities like exchanges, clearinghouses or payment systems, as well as standard utilities like electricity transmission networks. Continuation of service is essential, so risky utilities cannot be liquidated. The optimal regulatory contract minimizes the social cost among the contracts that steer the firm away from risky activities. It is simple and implemented with a capital (equity) adequacy requirement and a resolution mechanism when that requirement is breached. The social cost function is explicitly computed, and comparative statics can be simply derived.

## Disciplines

Business

## Publication Details

Rochet, J. & Roger, G. (2016). Risky utilities. Economic Theory, 62 (1), 361-382.

**Noname manuscript No.**
(will be inserted by the editor)

# Risky Utilities

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September 2015

**Abstract** We develop a theory of "risky utilities", i.e. private firms that manage an infrastructure for public service, and that may be tempted to engage in excessively risky activities, such as reducing maintenance expenditures (at the risk of provoking a break-down of the system) or in speculation (at the risk of incurring massive losses it cannot bear). These risky utilities include financial utilities like exchanges, clearinghouses or payment systems, as well as standard utilities like electricity transmission networks. Continuation of service is essential, so risky utilities cannot be liquidated. The optimal regulatory contract minimizes the social cost among the contracts that steer the firm away from risky activities. It is simple and implemented with a capital (equity) adequacy requirement and a resolution mechanism when that requirement is breached. The social cost function is explicitly computed and comparative statics can be simply derived.

**Keywords** moral hazard, dynamic contract, speculation, capital requirements.

JEL Classification: D82, D86, G28, L43.

We thank an anonymous referee for useful comments as well as seminar participants at UBC, Copenhagen Business School, the Reserve Bank of Peru, the Bank of Korea, the University of Melbourne, the University of NSW, the University of Queensland, the Australian National University, and conference participants at ESAM 2014 (Hobart), the European Sovereign Debt Crisis conference and the 2015 European Winter Finance Summit. This project has received funding from the European Research Council under the European Community Seventh Framework Programme (FP7/2007-2013) grant agreement 249415-RMAC and NCCR FinRisk (project Banking and Regulation.)

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## 1 Introduction

Utilities are private firms that maintain infrastructure for a public service. Classical examples are distribution networks for electricity, natural gas or water, or generators. Utilities are traditionally considered “safe” in several dimensions. They are often monitored by public authorities, who are supposed to ensure that the physical infrastructure is well maintained. They often benefit from the implicit guarantee of the government should they encounter financial difficulties. They are typically regarded as a safe investment. For example the US electricity company Con-Edison is renowned for having steadily increased its dividend over the last 40 years.[1]

Several scandals have recently altered this perception. The California rolling blackouts of 2000-2001 showed that utilities can fail in spectacular ways. Not only can they go bankrupt, as in the case of Pacific Gas and Electric, but power can altogether stop flowing to users. At the same time Enron’s downfall exposed speculative activities at the source of both its failure and the California crisis. These events are not unique: the 2003 American Northeast blackout affected an estimated 10 million people in Ontario and 45 million in eight US states. Its origin is attributed to a lack of pruning of trees, which interfered with the transmission lines. In 2009, a power line fell to the ground in Kilmore East (Victoria, Australia) and started a fire that killed 119 people. The ensuing settlement amounted to AUD 500 million; the transmission line had fallen because of a faulty conductor lacking a protective cap costing $10.

Moreover the Global Financial Crisis (GFC) of 2007-09 has renewed interest in the notion of “utility banking”, i.e. banking activities that are viewed as essential to the economy.[2] This notion has lead to several proposals by Volcker in the US (2010), Vickers in the UK (2011) and Liikanen in the EU (2012) for (i) separating or at least ring-fencing the “utility” activities of banks from speculative activities such as proprietary trading and (ii) designating some institutions as systemically significant and thus subject to enhanced regulatory scrutiny and special resolution processes.[3]

A consequence of the GFC was the adoption of special regulations for financial institutions whose interruption of service would entail large social costs. The Dodd-Frank Act introduced the notion of “Financial Utility”: financial infrastructures that are vital for the US economy, such as securities or derivatives ex-

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[1] Con Edison investor relations: http://investor.conedison.com/phoenix.zhtml?c=61493&p=irol-dividends.

[2] In his Guardian article “*Taming the financial casino. We need to restore narrow banking – to ensure that risky bets cannot again jeopardize the utility*”, of March 24, 2009, John Kay claimed: *“We attached a casino – proprietary trading activity by banks – to a utility – the payment system, together with the deposits and lending that are essential to the day-to-day functioning of the non-financial economy.”*

[3] Goodhart (2013) analyzes in detail why this ring-fencing may be difficult to implement. We will not explore this direction here.

changes, large-value payment systems and clearinghouses.[4] According to Paul Tucker (Deputy Governor, Bank of England) the consequence of the failure of such an institution is "mayhem", as he witnessed in 1987, when the Hong Kong Futures Exchange clearinghouse failed as a consequence of the stock crash.[5] It resulted in Hong Kong's futures market and its stock market closing also for a time; the stock market re-opened 45.5% lower. Such a drop erodes the savings and retirements accounts of large fractions of the population, and thereby affects their consumption and retirement decisions. It also typically engenders disruptive liquidity crises.

This article proposes a theory of the regulation of these "Risky Utilities". Most utilities are already subject to regulation, but its object is to curb their market power. A vast academic literature has studied this form of utility regulation starting with Hotelling (1938), Dupuit (1952) and expanded since by Baron and Myerson (1982), Sappington (1983) and Laffont and Tirole (1986) (see also Laffont and Tirole, 1993). Instead we lay the emphasis on the problem of risk management.

Our model is general enough to address both the maintenance problems of traditional utilities and the speculation problems of financial utilities. We label a risky utility any private company that manages an infrastructure for public service, and that may be tempted to reduce maintenance expenditures (at the risk of provoking a break-down) or to engage in speculative activities (at the risk of incurring massive losses it cannot bear). We focus on the "pure" utility problem and capture the notion of a risky utility by three simple features:

- the company can secretely engage in risky activities (lack of maintenance or speculation) that increase profit but may provoke huge losses (a catastrophe);
- shut-down would exert large negative externalities, so the firm cannot be liquidated and public authorities must intervene following the catastrophe;[6]
- the size of the firm is constant so that we abstract from the questions of investment policy. This fits public exchanges, clearinghouses and electricity transmission networks.

Public authorities have the power to regulate the company ex ante and restructure it ex post, should a catastrophe occur. The object of the article is to determine the best regulation contract. To this end we develop a continuous-time model of risk-taking under moral hazard in that is tractable enough to allow for a quasi-explicit solution. Comparative statics are then easy to derive.

A regulated firm (the agent) can engage in two types of socially wasteful activities: cash-flow diversion and risky activities (speculation) that improve short term profitability but may trigger large losses governed

[4] As of September 2014 eight entities in the U.S. have been designated Systemically important financial market utilities (SIFMU).

[5] Financial Times, 16 April 2012.

[6] A SIFMU is not subject to bankruptcy law and so cannot be liquidated; instead it is to be placed under receivership under the administration of the FDIC.

by a Poisson process. The firm is protected by limited liability. An incentive-compatible regulation contract deters both, and the optimal contract minimizes the social cost of regulation among incentive compatible contracts. This contract is very simple: it is a termination rule of the incumbent with on sale to new investors. That intervention is triggered when the value of the firm falls below a threshold; this is interpreted as, and implemented with, a minimal capital requirement. This threshold corresponds to the lowest continuation value that deters speculation. Equity here is not meant to absorb losses; it guarantees the owners of the firm have enough to lose to not speculate. The more efficient is diversion the more attractive is speculation, and so the more difficult it is to deter. Deterring speculation thus requires a higher equity threshold. So more efficient cash flow diversion induces a higher social cost (because of the option to speculate).

Our paper is related to four strands of the literature. First, we use the continuous time contracting techniques as developed by Biais et al (2007,2010), DeMarzo and Sannikov (2006) and Sannikov (2008). They are particularly appropriate in our context: the decision to speculate can be altered at any point in time, restructuring naturally corresponds to a stopping time and a large loss can arise at any moment with minute probabilities. The model can be viewed as an extension of DeMarzo, Livdan and Tschistyi (2013). We depart from this model in several ways; the most notable are: (i) a contract cannot be conditioned on an exogenous observable event (a crisis), so "relative performance" evaluation is not possible; (ii) we do not rely on public randomization for termination because of the very large losses. Second, we connect to the regulation of equity capital. Our optimal contract is implemented with a minimal equity requirement imposed on the firm, the purpose of which is to ensure the shareholder has enough at stake not to engage in excessive risk-taking. Furlong and Keeley (1989) establish that asset risk decreases when the capitalization of a bank increases. Milne (2002) observes that a bank's portfolio choice depends on its capitalization. Our model accords well with both; capital requirements induce the institution to choose the less risky path because breaching the capital requirement triggers restructuring and expropriation. In Diamond and Rajan (2000) bank capital acts as a buffer against credit losses and thus curtails bank runs by providing depositors with some insurance. Similarly in Tian, Yang and Zhang (2013) capital buffers protect the banks against contagion arising from credit losses. That capital may be long-term debt; not so in our model, where it has to be equity to deter speculation.

Third, the paper is related to the literature on financial structure and risk taking, as in Biais and Casamatta (1999). They model an agency problem with two actions however the goal is to determine the optimal financial structure of the firm without externalities. As in our paper, equity is necessary to overcome the risk-taking problem. We also connect to a more recent literature on bailouts. Zentefis (2014) shows the nature of the rescue matters: if the institution is burdened by excessively large repayments ex post (as a debtor, for example) it has incentives to default. In our model there is no default but early intervention that is final. Panageas (2010) and Kacperczyk and Schnabl (2013) show that an implicit guarantee, or more capital (respectively), enhances return volatility. This is empirically echoed in Mariathasan, Merrouche and Werger (2014). Our agent is protected by limited liability and would *always* like more risk, so an deterrent must be preemptive.

Section 2 presents the model. Section 3 characterizes incentive compatible regulation contracts and suggests a simple implementation. Section 4 studies the social cost function in details. We present a discussion in Section 5 and then conclude. All proofs are relegated to the Appendix.

## 2 Model

We adapt the model of DeMarzo, Livdan and Tschistyi (2013) to the case of an infrastructure providing a public service that must be continued in all circumstances. Operating cash flows follow the process

$$dx_t = \mu(a)dt + \sigma dZ_t - KdN_t, \tag{2.1}$$

where $a \in \{0, 1\}$ and $Z \equiv \{Z_t, \mathcal{F}_t; 0 < t < \infty\}$ is a standard Brownian motion associated with a filtration $\mathcal{F}_t$ on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$. The action $a$ governs the trend $\mu(\cdot)$ and the intensity of the Poisson process $N_t$ : $\mu(0) = \mu, \mu(1) = \mu + \Delta\mu$ and $\lambda(0) = 0, \lambda(1) = \Delta\lambda$. We assume $\mu > 0,\ \Delta\mu > 0$.

There are two sources of frictions. First, in the spirit of DeMarzo and Sannikov (2006) the operating cash flow at any moment $t$ can be diverted by the shareholder: a dollar diverted brings $\eta \leq 1$ dollars to the shareholder. Second, the agent can secretely engage in excessively risky ("speculative") activities that generate an additional cash-flow $\Delta\mu$ per unit of time but expose the firm to catastrophic losses $K$ that wipe it out.[7] For example, the firm sells (but does not buy) credit default swaps or options. Or an electricity network may save $\Delta\mu$ on its maintenance, and thereby expose itself to network failure governed by the Poisson process.

The government auctions off the right to operate that infrastructure among a pool of agents (potential investors/managers) who have limited wealth $\omega$. At any point in time the infrastructure is operated by a particular investor/manager: the shareholder. There are also passive investors who can participate in the financing of the infrastructure. All agents are risk-neutral and discount future payments at a common rate $r > 0$. The shareholder is subject to an exogenous shock (e.g. liquidity shock or investment opportunity) governed by an independent Poisson process of intensity $\delta$. Whenever hit by this shock the shareholder must divest; the associated stopping time is $\tau_L$. This maps well into the fact that investors in public infrastructure do not hold their assets forever, and that these divestments may occur randomly. Thus restructuring may be triggered either for exogenous reasons or for insufficient performance. The latter is the contractual restructuring

[7] These may be financial losses as experienced during the GFC or social losses associated with business interruption for a more traditional utility. Then the regulatory contract forces the shareholder to internalize these externalities.

associated with the stopping time $\tau_R$. Hence the stopping time $\tau = \tau_L \wedge \tau_R$: it is the minimum of either stopping time.

A regulation contract $\Xi$ specifies the flow of payment (dividends) $dL_t$ to the shareholder, as well as the termination rule represented by a stopping time $\tau$ and a terminal payment $w_\tau$. At date $\tau$ the firm is restructured at cost $\gamma$: the incumbent shareholder receives a payment $w_\tau$ and the firm is sold to a new shareholder. Since the environment is stationary the terms of the new regulation contract $\Xi = (L, \tau, w_\tau)$ remain the same. The objective of the government is to minimize the expected present value of the public funds that need to be expended in order to guarantee the continuity of the service provided by the infrastructure. A regulation contract is incentive compatible if it is designed in such a way that the shareholder never finds it optimal to divert cash, nor to engage in speculative activities.

## 3 The Optimal Contract

Following the recursive approach of Spear and Srivastava (1987) we can characterize any contract by the stochastic process $w$ describing the continuation payoff of the agent when the contract $\Xi$ is executed. Let

$$dH_t(\Xi, a) = dL_t(\Xi, a) + \eta[dx_t(a) - d\widehat{x}_t(a)]$$

denote the agent's consumption process under contract $\Xi$ and strategy $a$, where she reports $d\widehat{x}_t(a)$ and when the actual cash flow is $dx_t(a)$. Since the agent cannot save $dx_t(a) - d\widehat{x}_t(a) \geq 0$. The agent's continuation utility at date $t$ takes the form

$$w_t(\Xi, a) = \mathbb{E}^a \left[ \int_t^\tau e^{-r(s-t)} dH_s \right]. \tag{3.1}$$

hence total utility reads

$$U_t(\Xi, a) = \mathbb{E}^a \left[ \int_0^{t\wedge\tau} e^{-rs} dH_s \middle| \mathcal{F}_t \right] + e^{-rt} w_t(\Xi, a). \tag{3.2}$$

Drop the dependence on $\Xi$ for convenience; using the martingale representation theorem the utility $U_t(a)$ may also be written

$$\widehat{U}_t(a) = \mathbb{E}^a \left[ \int_0^t P_s[dN_s^a - \lambda(a)ds] \right] + \mathbb{E}^a \left[ \int_0^t (\beta_s^a/\sigma) dZs \right],$$

where $N_t^a$ is a Poisson process of intensity $\lambda(a)$. Incentive compatible contracts can be directly characterized by simple conditions on these sensitivity parameters $\beta_t$ and $P_t$. We now proceed to completely describe these conditions.

### 3.1 Incentive compatibility

Recall that the process $L$ of payments to the shareholder satisfies the limited liability constraints $dL_t \geq 0$ for all $t$, and $w_\tau \geq 0$. From the definition of $w_t$ this implies

$$w_t \geq 0, \ \forall t.$$

**Proposition 1** *No cash is diverted if and only if*

$$\beta_t \geq \beta \equiv \eta\sigma, \tag{3.3}$$

*and there is no speculation if and only if*

$$P_t \geq \frac{\beta_t}{\sigma}\frac{\Delta\mu}{\Delta\lambda}. \tag{3.4}$$

Combining these two conditions gives the necessary condition

$$P_t \geq \eta\frac{\Delta\mu}{\Delta\lambda} \equiv w_m$$

To deter the agent from diverting funds to her own use, the principal specifies a share $\beta_t/\sigma$ of cash flows that the agent can keep. Then she prefers (at least weakly) receiving $\beta_t dZ_t$ from the principal to appropriating the usable fraction $\eta$ of $\sigma dZ_t$. Similarly, engaging in speculation generates an additional $\Delta\mu$ but may trigger the penalty $P_t$. Incentive compatibility requires that the flow expected loss from speculation ($\Delta\lambda P_t$) be at least as large as the flow expected gain ($\eta\Delta\mu$).

### 3.2 Characterization of the optimal contract

Our first Proposition outlines the set of incentive compatible contracts. Now we turn to the contract that minimizes the social cost function among all incentive compatible contracts. This social cost function is explicitly defined in Section 4 but we can already characterize the optimal contract as follows.

**Proposition 2** *The optimal contract is such that*

- $L_t \equiv 0$ *(compensation is deferred to date $\tau$).*
- $\beta_t \equiv \beta$ *(minimum cash flow sensitivity that prevents cash diversion);*
- $P_t \equiv w_t$ *(the shareholder is wiped out in case of a catastrophe);*[8]
- $\tau = \tau_L \wedge \tau_R$, *where* $\tau_R = \inf\{t | w_t \leq w_m\}$ *(termination occurs at the earliest of the exogenous retirement and the regulatory intervention threshold for insufficient performance.)*

These conditions are easy to interpret. Since government and shareholders have the same discount factor it never helps to disburse any cash in the form of early (that is, before termination) payments $dL_t$. This is an extreme form of back-loading payments, in order to provide maximum incentives at minimum cost. Moreover it is optimal for the government to allow for the smallest fraction $\beta_t$ of the volatile component of the cash flow $dx_t - \mathbb{E}[dx_t] = \sigma dZ_t$ to be left to the shareholder. The limited liability constraint on $w_t$ implies that $P_t \leq w_t$; thus imposing a higher penalty may only trigger earlier restructuring without altering the shareholder's incentives. Hence, along the optimal path, $w_t$ is subject to the dynamics

$$dw_t = rw_t dt + \beta dZ_t.$$

[8] Since catastrophes do not occur along the equilibrium path in this model, any $P_t \geq w_m$ is also optimal.

The incentives are maximized when $P_t \equiv w_t$: the shareholder must be wiped out after a catastrophe. Any further penalty would violate limited liability, thus $w_t \geq w_m$ is necessary to preserve incentive compatibility. The firm must be restructured or recapitalized when $w_t$ reaches $w_m$.

### 3.3 Implementation of the optimal contract

In line with Biais , Mariotti, Plantin and Rochet (2007, hereafter BMPR) we implement the optimal contract using a well-selected financing and cash management policy.[9] The fundamental principle underlying this implementation is that the firm is required to maintain cash reserves

$$m_t \equiv \frac{w_t}{\eta}$$

that stay proportional to the continuation payoff $w_t$. At date 0 the winning bidder (who becomes the shareholder) invests $\omega$. The firm issues riskless debt (to the passive investors) paying a constant coupon $\mu$, which is guaranteed by the government; the market value of this debt is therefore $\mu/r$. The government initially injects

$$m_0 - \omega + I + \gamma - \frac{\mu}{r}$$

in exchange for a a fraction $1-\eta$ of the shares (outside equity).[10] Here $m_0$ is the initial level of cash reserves and $I \geq 0$ is the once-and-for-all investment necessary to start the infrastructure, and $\gamma$ is the cost of organising the auction. The balance sheet at date $t$ is shown below.

| Productive Assets<br>A | Debt<br>D |
|---|---|
| Cash reserves<br>$m_t$ | Inside equity<br>$\eta v_t$<br>Outside equity<br>$(1-\eta)v_t$ |

The value of the productive assets is the expected present value $\mu/r$ of the cash flow and thus coincides with the value of the debt $D$. This implies that $m_t \equiv v_t$: the total value of the equity is always equal to the cash reserves, because all players have the same discount rate.

Under the optimal contract there is no speculation so the cash reserves of the firm follow the dynamics

$$dm_t = \underbrace{rm_t dt}_{interest} + \underbrace{\mu dt + \sigma dZ_t}_{earnings} - \underbrace{\mu dt.}_{coupon}$$

[9] The implementation is not unique. DeMarzo and Sannikov (2006) suggest an implementation using credit lines instead.

[10] This is a sleeping participation: the government is not actively engaged in the management of the firm – except in case of restructuring.

Therefore the process

$$dm_t = \frac{dw_t}{\eta} = rm_t dt + \sigma dZ_t \tag{3.5}$$

is also a discounted martingale. Restructuring takes place at time $\tau = \tau_L \wedge \tau_R$, where $\tau_R$ is the first time the cash reserves fall below $\Delta\mu/\Delta\lambda$ – recall $m_t = w_t/\eta$. The government removes the management and looks for a new shareholder. Nonetheless there is no breach of contract: termination is part of the contract and the shareholder receives the contractual payment $w_\tau$. The net injection of public funds at that time is thus contingent on $w_\tau$. However the social cost of restructuing is independent of these transfers, even if $\eta < 1$; it is simply $\gamma > 0$, i.e. the cost of finding a new shareholder.

The total value $v_t$ of the equity of the firm, including the shares accruing to the government, is just equal to $m_t$. The private shareholder holds a fraction $\eta$ of the equity

$$w_t = \eta v_t$$

while the government's participation is $(1 - \eta)v_t$. Hence the stopping time can also be regarded as the first time the value of the firm's equity falls below $v_m \equiv \Delta\mu/\Delta\lambda$. This is a capital adequacy rule.

*Remark 1* The optimal contract uses a combination of debt and equity to mitigate the two frictions. Debt solves the cash diversion problem by appropriating expected earnings $\mu dt$. Equity is used to prevent speculation: the minimum capital requirement ensures that the shareholder has enough "skin in the game" to not engage in excessively risky activities.

*Remark 2* Our result is also related to the efficiency wage model of Shapiro and Stiglitz (1984). They study firms-workers relationships when workers can "shirk" but can be detected with some exogenous probability, in which case they are fired. The efficiency wage is the minimum wage that deters workers from shirking. In our model, the analogue of the efficiency wage is the minimum market value of equity below which the firm starts engaging in excessive risk taking. Then its shareholders are "fired".

From now on we use the equity value $v_t$ ($\equiv m_t$) as state variable instead of $w_t$ ($\equiv \eta v_t$). This variable determines the expected cost of public intervention through the cost function $C(v)$ that we now study in detail.

## 4 The social cost of public intervention

Here we analyze in detail the determinants of the social cost of restructuring the firm, for which we need to characterize the social cost function. The cost of public intervention is related to the optimal regulation contract through the recursive formulation

$$C(v) = [\gamma + C(v_0)]\, \mathbb{E}\left[e^{-r\tau}|v\right], \tag{4.1}$$

where $\tau = \tau_L \wedge \tau_R$, $\tau_L$ is exogenous, independent of $Z$ and follows a Poisson process with intensity $\delta$ and $\tau_R$ is the first time the equity of the firm falls below the threshold $v_m$:

$$\tau_R = \inf \{t | v_t \leq v_m\}.$$

Finally the value $v_t$ of the firm follows a discounted martingale:

$$dv_t = rv_t dt + \sigma dZ_t. \tag{4.2}$$

Transfers between the government and shareholders (incumbent and new) do not appear in the social cost formula (4.1). Since the utility is never discontinued, expected future cash flow amounts to a constant $\mu/r$ that can be ignored (this is paid out to debt holders). Therefore social costs at any point in time $t$ are simply the expected present value of future restructuring costs. The recursive formulation expresses it as the sum of the expected present values of the cost of the next restructuring $\gamma e^{-r\tau}$ and the continuation cost $C(v_0)e^{-r\tau}$. The government is constrained by the limited wealth $\omega = \eta v_0$ of new investors.

### 4.1 Characterization of the social cost function

We begin by outlining a complete characterization of the function $C(v)$ under the optimal contract. From the properties of the optimal contract, $C(\cdot)$ satisfies the following differential equation

$$(r+\delta)C(v) = rvC'(v) + \frac{\sigma^2}{2}C''(v) + \delta[C(v_0)+\gamma], \quad v \geq v_m \tag{4.3}$$

with boundary conditions

$$C(v_m) = C(v_0) + \gamma \tag{4.4}$$

and

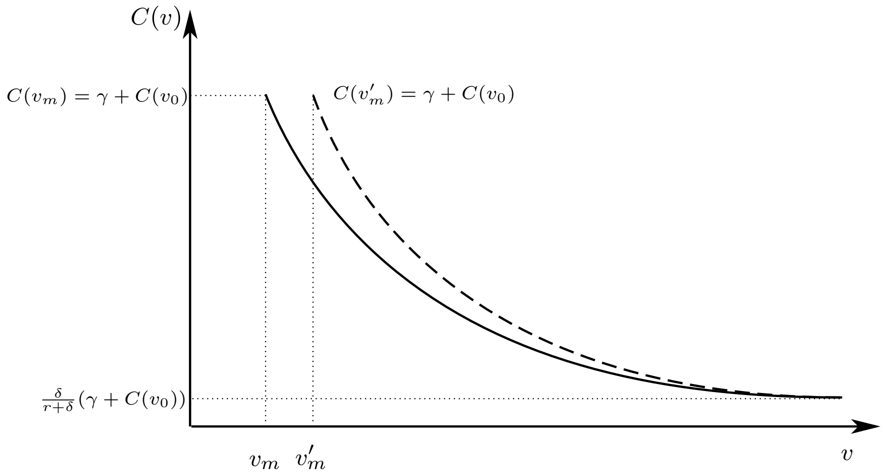
$$\lim_{v\to\infty} C(v) = \frac{\delta}{r+\delta}\left[C(v_0)+\gamma\right]. \tag{4.5}$$

The first boundary condition is the optimal termination condition. When $v_t < v_m$ speculation can no longer be prevented (by Condition (3.4)). Therefore the firm must be restructured as soon as $v_t = v_m$; the government incurs a cost $\gamma$ and resets the firm's continuation value at $v_0$. The second condition comes from the fact that

$$C(v) \geq \mathbb{E}\left[e^{-r\tau_L}\left(C(v_0)+\gamma\right)\right],$$

that is, the social cost is at least the cost of occasional (exogenously given) restructures when shareholders divest because of their exogenous shock. Compulsory restructuring may arise before $\tau_L$ but its probability converges to zero when $v \to \infty$. Because

$$\mathbb{E}\left[e^{-r\tau_L}\left(C(v_0)+\gamma\right)\right] \equiv \frac{\delta}{r+\delta}\left[C(v_0)+\gamma\right],$$



The social cost function $C(v)$ parametrized by $v_m$ or $v'_m$.

Condition (4.5) follows.

These boundary conditions are not standard: the function $C(v)$ appears on both sides of (4.4) and (4.5). It is nonetheless possible to show there exists a unique solution $C(v)$.

**Proposition 3** *When $\delta \le 3r$ there is a unique solution to the homogenous equation*

$$(r+\delta)A(v) = rvA'(v) + \frac{\sigma^2}{2}A''(v)$$

*such that $A(0) = 1$ and $A(\infty) = 0$. Then, the function*

$$C(v) = \frac{\gamma}{A(v_m) - A(v_0)}\left[\frac{\delta}{r}A(v_m) + A(v)\right] \tag{4.6}$$

*is the unique solution of the differential equation (4.3) with boundary conditions (4.4) and (4.5). It is decreasing and convex on $[v_m, \infty)$.*

The function $A(\cdot)$ can be expressed as a linear combination of confluent hypergeometric functions of the first kind $M(a, b; z)$:[11]

$$A(v) = M\left(-\frac{1}{2}\left(1+\frac{\delta}{r}\right), \frac{1}{2}; -\frac{rv^2}{\beta^2}\right) - \frac{2v\sqrt{r}}{\beta}\frac{\Gamma\left(\frac{3}{2}+\frac{\delta}{2r}\right)}{\Gamma\left(1+\frac{\delta}{2r}\right)}M\left(-\frac{\delta}{2r}, \frac{3}{2}; -\frac{rv^2}{\beta^2}\right),$$

where $\Gamma(\cdot)$ denotes the Gamma function. Function $C(v)$ is depicted in Figure 1.

The convexity of $C(v)$ is the reason why it is indeed optimal to set the sensitivity $\beta_t$ of the shareholder's continuation value $w_t$ to its minimum value: $\beta_t \equiv \eta\sigma$. In addition, any early payment to the agent $(dL_t)$ would decrease $v_t$ and therefore the survival probability of the firm. Similarly any increase in the penalty $P_t$ beyond $w_m$ increases the probability of restructure; more precisely it triggers a restructure before it is actually necessary.

[11] See Abramowitz and Stegun (1964).

### 4.2 Properties of the social cost function

The explicit characterization of the social cost function allows to derive easily several comparative statics results:

**Proposition 4** *The social cost of public intervention*

1. *increases with the minimum capital requirement* $v_m = \Delta\mu/\Delta\lambda$*;*
2. *decreases with the wealth* $\omega$ *of potential shareholders;*
3. *increases with the efficiency* $\eta$ *of the cash diversion technology;*
4. *is proportional to the restructuring cost* $\gamma$*;*

Some of these comparative statics deserve commentary. It is easy to see that the social cost decreases with the initial equity injection $v_0 = \omega/\eta$, which explains why $C(\cdot)$ decreases in $\omega$ and increases in $\eta$. That $C(v)$ increases with $v_m$ may not be so immediate. These comparative statics are connected, and their impact relates to Proposition 2. Increasing $v_m$ simply increases the frequency of costly restructures.

## 5 Discussion

The optimal regulation contract can be implemented using an appropriate combination of debt and equity with an appropriate termination rule. As in other papers, debt has a disciplining effect: it is used to extract the firm's free cash flow, which prevents cash diversion. But it is not sufficient and leaves open the problem of speculation, so equity is necessary too. It takes the form of a minimal equity requirement, which guarantees that the shareholder keeps enough at stake to not engage in excessive risk-taking. This equity requirement is complemented with restructuring (which includes expropriation and compensation at market value of the shareholder) that is triggered every time the capital requirement is violated.

Thus the equity requirement has a quite a different role than the "buffer against losses" often advocated in the banking regulation literature. Instead of absorbing losses and reducing the cost (and frequency) of public intervention, a higher capital requirement increases them! The reason is that a higher requirement $v_m$ corresponds to a higher expected return on speculative activities $\Delta\mu/\Delta\lambda$. Then restructuring is bound to occur more frequently for any bounded wealth $\omega$ (to prevent speculation). It would be cheaper in the short run for the government to ignore the breach of capital requirement (and possibly only restructure upon insolvency). But this violates incentive compatibility and therefore is socially too costly. So the capital requirement can also be seen as necessary to prompt early corrective action. This action must be drastic here, since social losses can be very large.

Our resolution mechanism is termination and sale to a new shareholder. It very much differs from a bailout: termination occurs not because of financial distress but to preserve incentive compatibility. Yet it guarantees continuation of service, as is socially desirable for SIFMUs and many other utilities. This differs from the proposals of Tucker (2014), who advocates orderly wind-down of financial utilities in distress. If these financial utilities are indeed essential, wind-down is not credible and that regulation is toothless.

Monitoring is a standard remedy to moral hazard. Here one has to be careful as to *what* is monitored. Monitoring that somehow results in reducing the change $\Delta\mu$ in the drift is uniformly positive: it reduces the threshold $v_m$ by curtailing the incentives to engage in risky activities. In contrast, monitoring to reduce the incidence of catastrophes $\Delta\lambda$ is uniformly bad(!). It increases $v_m$ in that it is a license to speculate: a large loss is even less likely. An immediate implication of this model in terms of risk management is that, to the extent it is possible, it is better to reduce the magnitude of losses ($K$) than their frequency $\Delta\lambda$.

## 6 Conclusion

We have characterized the optimal regulation contract for a "risky utility" in a dynamic model of risk-taking under moral hazard. The model is relevant for a broad range of applications, ranging from standard utilities to the newly designated financial utilities. The emphasis is laid on the survival risk of these businesses, and on the externalities their failure (either financial or operational) generates.

Regulation is needed to alleviate two frictions. Shareholders can divert some of the earnings to their benefit. They can also engage in excessively risky activities to increase those earnings in the short run at the expense of catastrophic losses (in the longer term).

The optimal contract can be implemented by an appropriate mix of debt and equity, and a stringent termination rule. The equity requirement plays a very different role than in standard models of banking regulation. It is not there to absorb losses but instead to discipline the firm and to trigger "prompt corrective action" from the government. Preventing regulatory forbearance is thus of primary importance for risky utilities.

Appendix

## A Some background

In the main text we set aside some technicalities that we develop here. The action $a$ impacts the drift $\mu(a)dt$ and the intensity $\lambda(a)$ of the Poisson process $N_t$ of losses $K$. In so doing it generates a probability measure over the path of $x_t$; where not explicitly stated all expectations are nonetheless taken with respect to that measure. Correspondingly, a contract involves a $\mathcal{F}_t^N$ -adapted cumulative payment $L_t$ to the shareholder and a $\mathcal{F}_t^N$ -stopping time $\tau_R$.

## B Proofs

*Proof (of Proposition 1)* Given the contract $\Xi$ and information at $t$, an agent's utility under any strategy $a$ is

$$\begin{aligned} U_t(\Xi, a) &= \mathbb{E}^a \left[ \int_0^\tau e^{-rs} dH_s(\Xi, a) | \mathcal{F}_t \right] \\ &= \mathbb{E}^a \left[ \int_0^\tau e^{-rs} dL_s(a) + \eta \left( dx_s(a) - d\widehat{x}_s(a) \right) | \mathcal{F}_t \right] \end{aligned}$$

where $dH_t(\Xi, a) = dL_t(\Xi, a) + \eta(dx_t(a) - d\widehat{x}_t(a))$. We now drop the dependence on $\Xi$ for convenience. At any $t$, the continuation utility of an agent is

$$w_t(a) = E^a \left[ \int_t^\tau e^{-r(s-t)} dH_s(a) | \mathcal{F}_t \right] \tag{B.1}$$

so $U_t(a)$ rewrites

$$U_(a) = \mathbb{E}^a \left[ \int_0^{t \wedge \tau} e^{-rs} dH_s(a) | \mathcal{F}_t \right] + e^{-rt} w_t(a)$$

Because $U_t(a)$ is a martingale, for some process $P_t$, utility may also be expressed as

$$U_t(a) = U_0(a) + \int_0^{t\wedge\tau} e^{-rs} \frac{\beta_t}{\sigma} dZs - \int_0^{t\wedge\tau} e^{-rs} P_s (dN_s(a) - \lambda(a) ds)$$

where $M_t^a = N_t^a - \int_0^t \lambda(a) ds$. Suppose the agent follows $a_s = 1,\ s < t$ and then reverts to $a_s = 0$. then

$$\begin{aligned} U_t(1) &= \mathbb{E}^{a=1} \left[ \int_0^{t\wedge\tau} e^{-rs} dH_s(1) | \mathcal{F}_t \right] + e^{-rt} w_t(0) \\ &= U_t(0) + \int_0^{t\wedge\tau} e^{-rs} \left( dH_s(1) - dH_s(0) \right) \\ &= U_0(0) - \int_0^{t\wedge\tau} e^{-rs} P_s dM_s^0 + \int_0^{t\wedge\tau} e^{-rs} \left( dH_s(1) - dH_s(0) \right) \\ &= U_0(0) - \int_0^{t\wedge\tau} e^{-rs} P_s dM_s^1 - \int_0^{t\wedge\tau} e^{-rs} P_s (\lambda(1) - \lambda(0)) ds + \int_0^{t\wedge\tau} e^{-rs} \left( dH_s(1) - dH_s(0) \right) \\ &= U_0(0) - \int_0^{t\wedge\tau} e^{-rs} P_s dM_s^1 - \int_0^{t\wedge\tau} e^{-rs} P_s \lambda(1) ds + \int_0^{t\wedge\tau} e^{-rs} \left( dH_s(1) - dH_s(0) \right) \end{aligned}$$

The first line is a definition, the second one expresses the gains from departing from $a = 0$ before time $t$, the third one is the martingale representation of the second one. The fourth equality teases out the penalties associated with the strategy $a = 1$ and the last one is the corresponding computation. In the last line the term

$$U_0(0) - \int_0^{t\wedge\tau} e^{-rs} P_s dM_s^1$$

is a martingale: $\mathbb{E} \left[ \int_0^{t\wedge\tau} e^{-rs} P_s dM_s^1 \right] = 0$. So the drift of $U_t(a)$ is given by the sign of

$$\int_0^{t\wedge\tau} e^{-rs} \left[ (dH_s(1) - dH_s(0)) - P_s \lambda ds \right],$$

hence we require

$$P_t dt \geq \frac{dH_t(1) - dH_t(0)}{\lambda}$$

to deter speculation, where

$$\begin{aligned}\frac{dH_t(1) - dH_t(0)}{\lambda} &= \frac{1}{\lambda}\left[dL_t(1) - dL_t(0) + \beta\left(dx_t(1) - dx_t(0) - d\widehat{x}_t(1) - d\widehat{x}_t(0)\right)\right] \\ &= \eta\left[dZ_t - dZ_t + \Delta\mu\right] \\ &= \eta\Delta\mu\end{aligned}$$

When there is no speculation the dynamics of the utility $U_t(a)$ are given by

$$dU_t(a) = dH_t(a) + dw_t - rw_t dt$$

and those of $\widehat{U}_t(a)$ by

$$P_t[dN_t - \lambda(a)dt] + \frac{\beta_t}{\sigma} dZ_t$$

Equating these two and re-arranging,

$$dw_t = rw_t dt - dH_t + \frac{\beta_t}{\sigma} dZ_t - P_t[dN_t - \lambda(a)dt].$$

Now substituting for the definition of $dH_t, w_t$ is a supermartingale only if

$$\frac{\beta_t}{\sigma} \geq \eta,$$

which is (3.3). This condition mirrors DeMarzo and Sannikov (2006). Combining

$$P_t \geq \eta \frac{\Delta\mu}{\Delta\lambda}$$

with (3.3) binding one has (3.4), which is feasible only when $w_t \geq w_m$ by limited liability. When these two constraints are satisfied the agent prefers to not speculate and to not divert funds either. □

*Proof (of Proposition 2)* Setting $P_t > w_m$ is neutral on the firm's incentives whether to engage in speculation. However recall that

$$\tau = \tau_L \wedge \tau_R = \tau_L \wedge \inf\{t | w_t = P_t\}$$

for any $P_t$, and where the equality owes to the limited liability constraint $P_t \leq w_t$. Clearly $\tau_R$ is decreasing in $P_t$ so that $\tau$ is at least weakly decreasing. The lowest penalty $P_t$ that is compatible with incentive compatibility is $\omega$. Because government and shareholder discount the future at the same rate $r$, there is no cost in substituting payments for an increase in the continuation value $w_t$. There is a strict benefit to doing so since $\tau_R = \inf\{t | w_t = P_t\}$. From the dynamics of the continuation value under an incentive compatible contract

$$dw_t = rw_t dt - dL_t + \beta_t dZ_t,\ w_t \geq w_m$$

one sees that decreasing $dL_t$ correspondingly shifts the trajectory of $w_t$. So it is in the government's interest to set $dL_t \equiv 0$. Then under an incentive compatible contract the agent's utility is

$$dw_t = rw_t dt + \beta_t dZ_t,\ w_t \geq w_m.$$

As we show below, social cost is a convex function of $\omega$. Hence the optimal contract involves $\beta_t \equiv \eta\sigma$, the minimum value that satisfies the incentive compatibility condition. □

*Proof (of Proposition 3)* From (4.3), the function $C$ takes the form

$$(r+\delta)C(v) = \delta\left[\gamma + C(v_0)\right] + c_0 H_0(v) + c_1 H_1(v)$$

where $\delta\left[\gamma + C(v_0)\right]$ is a particular solution of the second-order ODE and $(H_0, H_1)$ are basis of solutions for the homogenous equation

$$(r+\delta)H(v) = rvH'(v) + \frac{\sigma^2}{2}H''(v)$$

with

$$\begin{aligned} H_0(0) &= 1 = H_1'(0) \\ H_1(0) &= 0 = H_0'(0). \end{aligned}$$

The confluent hypergeometric function of the first kind $M(a,b;z)$ is the unique solution the confluent hypergeometric differential equation (also called Kummer's equation)

$$aM(z) = (b-z)M'(z) + zM''(z); \quad M(0) = 1, \ M'(0) = \frac{a}{b} \tag{B.2}$$

In the next two Lemmata we construct the basis functions $H_0$ and $H_1$ and show each solves Kummer's equation.

**Lemma 1** $H_0(v) = M\left(-\frac{1}{2}\left(1+\frac{\delta}{r}\right), \frac{1}{2}; -\frac{rv^2}{\beta^2}\right)$

*Proof* Differentiate:

$$\begin{aligned} H_0'(v) &= -\frac{2rv}{\beta^2}M' \\ H_0''(v) &= -\frac{2r}{\beta^2}M' + \frac{4r^2v^2}{\beta^4}M'' \end{aligned}$$

So

$$rvH_0' + \frac{\beta^2}{2}H_0'' = -\left(\frac{2r^2v^2}{\beta^2} + r\right)M' + \frac{2r^2v^2}{\beta^2}M'' \tag{B.3}$$

and (B.2) becomes

$$-\left(\frac{1}{2} + \frac{\delta}{2r}\right)M = -\frac{rv^2}{\beta^2}M'' + \left(\frac{1}{2} + \frac{rv^2}{\beta^2}\right)M',$$

where $a = -(1/2 + \delta/2r)$ and $b = 1/2$. Hence by (B.3),

$$\underbrace{rvH_0' + \frac{\beta^2}{2}H_0''}_{=(r+\delta)H_0} = \underbrace{-2r\left[\left(\frac{rv^2}{\beta^2} + \frac{1}{2}\right)M' - \frac{rv^2}{\beta^2}M''\right]}_{=-2r\cdot aM_0}$$

The proof is complete once we have noted that $H_0(0) = M(0) = 1$ and $H_0'(0) = 0$. □

**Lemma 2** $H_1(v) = v \cdot M\left(-\frac{\delta}{2r}, \frac{3}{2}; -\frac{rv^2}{\beta^2}\right)$

*Proof* As in the proof of Lemma 1, differentiate

$$\begin{aligned} H_1' &= M - \frac{2rv^2}{\beta^2}M' \\ H_1'' &= -\frac{6rv}{\beta^2}M' + \frac{4rv^3}{\beta^4}M'' \end{aligned}$$

So that the RHS of the elementary differential equation writes

$$rvH_0' + \frac{\beta^2}{2}H_0'' = -\left(3rv + \frac{2r^2v^3}{\beta^2}\right)M' + \frac{2r^2v^3}{\beta^2}M'' + rvM_1 \tag{B.4}$$

and (B.2) reads

$$-\frac{\delta}{2r}M = -\frac{rv^2}{\beta^2}M'' + \left(\frac{3}{2} + \frac{rv^2}{\beta^2}\right)M',$$

where $a = -\delta/2r$ and $b = 3/2$. Therefore by (B.4),

$$\underbrace{rvH_1' + \frac{\beta^2}{2}H_1''}_{=(r+\delta)H_1} = \underbrace{-2rv\left[\left(\frac{rv^2}{\beta^2} + \frac{3}{2}\right)M\prime - \frac{rv^2}{\beta^2}M'' - \frac{M}{2}\right]}_{=-2rv\cdot aM_1}$$

and we note that $H_1(0) = 0, H_1'(0) = M(0) = 1$. $\square$

The functions $H_0, H_1$ and the boundary conditions (4.4) and (4.5) together give us a determination for the coefficients $c_0, c_1$. Condition (4.5) (i.e. $\lim_{v\to\infty} C(v) = \delta[C(v_0) + \gamma]/(r + \delta)$) implies

$$\lim_{v\to\infty}\left[H_0(v) + \frac{c_1}{c_0}H_1(v)\right] = 0 \tag{B.5}$$

directly from the definition of $C(v)$. So one has

$$\frac{c_1}{c_0} = -\lim_{v\to\infty}\frac{H_0(v)}{H_1(v)} \equiv c,$$

The parameter $c$ can be explicitly computed: any confluent hypergeometric function $M(a, b; z)$ can be expressed as

$$M(a, b; z) = \frac{\Gamma(b)}{\Gamma(b-a)}(-z)^{-a}\left[1 + O(|z|^{-1})\right], \quad z < 0$$

where $\Gamma(\cdot)$ is the Gamma function (see Abramovitz and Stegun (1964), Chapter 13, Theorems 13.1.4 and 13.1.5). Forming the ratio of $H_0$ and $H_1$ and simplifying yields

$$c = -\frac{\Gamma(1/2)}{\Gamma(3/2)}\frac{\Gamma(3/2 + \delta/2r)}{\Gamma(1 + \delta/2r)}\frac{\sqrt{r}}{\beta}$$

and since $\Gamma(1/2) = \sqrt{\pi}$ and $\Gamma(3/2) = (1/2)\sqrt{\pi}$,

$$c = -2\frac{\sqrt{r}}{\beta}\frac{\Gamma(3/2 + \delta/2r)}{\Gamma(1 + \delta/2r)} < 0.$$

confirming that limit exists. Condition (4.4) gives:

$$\frac{\delta}{r+\delta}[\gamma + C(v_0)] + c_0\left[H_0(v_m) + cH_1(v_m)\right] = \gamma + \frac{\delta}{r+\delta}[\gamma + C(v_0)] + c_0\left[H_0(v_0) + cH_1(v_0)\right],$$

which simplifies for $c_0$ as

$$c_0 = \frac{\gamma}{H_0(v_m) - cH_1(v_m) - (H_0(v_0) - cH_1(v_0))}$$

in terms of the functions now known $H_0, H_1$ and exogenous parameters only.

To complete the proof, observe from (4.6) that $C'(v)$ and $C''(v)$ are proportional to $A'(v)$ and $A''(v)$, hence $C(v)$ solves the equation

$$(r+\delta)C(v) = rvC'(v) + \frac{\sigma^2}{2}C''(v) + \frac{(r+\delta)\gamma}{A(v_m) - A(v_0)}\frac{\delta}{r}A(v_m)$$

and moreover

$$C(v_0) = \frac{\gamma}{A(v_m) - A(v_0)}\left[\frac{\delta}{r}A(v_m) + A(v_0)\right]$$

and

$$C(\infty) = \frac{\gamma}{A(v_m) - A(v_0)}\left[\frac{\delta}{r}A(v_m)\right]$$

It remains to check that

$$\begin{aligned}\delta[C(v_0) + \gamma] &= \frac{\delta\gamma}{A(v_m) - A(v_0)}\left[A(v_m) - A(v_0) + \frac{\delta}{r} + A(v_0)\right] \\ &= \frac{(r+\delta)\gamma}{A(v_m) - A(v_0)}\frac{\delta}{r}A(v_m)\end{aligned}$$

and

$$\begin{aligned} C(v_m) &= \frac{\gamma}{A(v_m) - A(v_0)} \left( \frac{\delta}{r} + 1 \right) A(v_m) \\ &= C(v_0) + \gamma \end{aligned}$$

Finally,

$$\begin{aligned} C(\infty) &= \frac{\gamma}{A(v_m) - A(v_0)} \frac{\delta}{r} A(v_m) \\ &= \frac{\delta}{r+\delta} [C(v_0) + \gamma] \end{aligned}$$

so indeed we can write (B.5)

$$\begin{aligned} C(v) &= \frac{\delta}{r+\delta} \left[ \gamma + C(v_0) \right] + c_0 \left[ H_0(v) + cH_1(v) \right] \\ &= \frac{\delta}{r+\delta} \left[ \gamma + C(v_0) \right] + c_0 A(v) \end{aligned} \tag{B.6}$$

where $c_0$ is

$$c_0 \equiv \frac{\gamma}{A(v_m) - A(v_0)}. \tag{B.7}$$

Since we can write

$$A(v) = H_0(v) + cH_1(v)$$

the boundary condition

$$\begin{aligned} A(v) &= H_0(0) + cH_1(0) \\ &= M\left( -\frac{1}{2}\left(1 + \frac{\delta}{r}\right), \frac{1}{2}; 0 \right) + c0 \cdot M\left( -\frac{\delta}{2r}, \frac{3}{2}; 0 \right) \\ &= 1 \end{aligned}$$

is immediately verified since $M(a,b;0) = 1$. For the second boundary we use the series

$$\begin{aligned} \frac{M(a,b;z)}{\Gamma(b)} &= \frac{e^{-i\pi a} z^{-a}}{\Gamma(b-a)} \left[ \sum_{n=0}^{R-1} \frac{(a)_n (1+a-b)_n}{n!} (-z)^{-n} \right] \\ &+ \frac{e^z z^{a-b}}{\Gamma(a)} \left[ \sum_{n=0}^{S-1} \frac{(b-a)_n (1-a)_n}{n!} z^{-a} \right] \end{aligned}$$

where $R, S$ are arbitrary positive integers and $(a)_n = a(a+1)...(a+n-1)$. With this the first term becomes

$$\begin{aligned} A(v) &= \left[ \frac{\Gamma(1/2)}{\Gamma(1+(\delta/2r))} - 2\frac{\Gamma(3/2)}{\Gamma(1+(\delta/2r))} \right] \left[ \left( \frac{rv^2}{\beta^2}^{\frac{r+\delta}{2r}} \right) + \frac{\delta}{4r}\left(1 + \frac{r}{\delta}\right) \left( \frac{rv^2}{\beta^2}^{\frac{1}{2}\frac{r}{\delta}-1} \right) \right] \\ &= 0 \end{aligned}$$

since $\Gamma(1/2) = 2\Gamma(3/2)$. The following terms can be shown to be of order no higher than $1 - 2a - 2 = -1(+\delta/r) < 0$ for the function $H_1(v)$ and no higher than $\frac{1}{2}(1+\delta/r) - 2$ for the function $H_0(v)$, hence the sufficient condition $\delta < 3r$.

**Lemma 3** *The function $A : \mathbb{R}_+ \mapsto \mathbb{R}$ is decreasing convex.*

*Proof* Since $A'(v) = H_0'(v) + cH_1'(v), A'(0) = 0 + c < 0$, so $A(v)$ is indeed decreasing in $v$ starting at $v = 0$. Furthermore, by

$$(r+\delta)A(v) = rvA'(v) + \frac{\sigma^2}{2}A''(0) \tag{B.8}$$

at $v = 0$

$$(r+\delta) = 0 + \frac{\sigma^2}{2}A''(0) > 0$$

Suppose now that $A(v)$ is not monotone. We rule out all cases in turns. First a local maximum with $A(v_1) > 0$ is impossible for then we must have $A(v_1) > 0, A'(v_1) = 0$ and $A''(v_1) < 0$, which contradicts (B.8). Second, a local minimum $v_2$ with $A(v_2) > 0$ is also impossible: at $v_2, A''(v_2) > 0$ and so there must be a local maximiser $v_3$ with $A(v_3) > 0$; we just ruled that out. Third, there cannot be an inflexion point with $A(v_1) > 0$ for then $A''(v_1) = 0$, which is again impossible by (B.8). Fourth, it cannot reach a local minimum $v_3$ where $A(v_3) < 0$, for then we must have $A(v_3) < 0, A'(v_1) = 0$ and $A''(v_3) > 0$. Again this is impossible by (B.8). Fifth, an inflexion point below 0 is impossible for then $A''(v_1) = 0$. Last, a local maximum with $A(v_4) < 0$ can also be ruled out: if so, there must be a local minimum with $A(v_5) < 0$, which was just shown to be impossible. $\square$

With (B.7) the social cost function reads

$$C(v) = \frac{\delta}{r+\delta}[\gamma + C(v_0)] + \frac{\gamma}{A(v_m) - A(v_0)} A(v).$$

where $A(v)$ is decreasing convex and $C(v_0)$ is a number. Therefore, from (B.7) again,

$$c_0 = \frac{\gamma}{A(v_m) - A(v_0)} > 0$$

since $v_0 > v_m$, and it follows that $C(v)$ is also decreasing convex. Finally together both this definition and the boundary condition (4.4) tell us that

$$\gamma + C(v_0) = \frac{r+\delta}{r} \frac{\gamma}{A(v_m) - A(v_0)} A(v_m)$$

substituting in the definition of $C(v)$ then yields

$$C(v) = \frac{\gamma}{A(v_m) - A(v_0)} \left[ \frac{\delta}{r} A(v_m) + A(v) \right]$$

as claimed. $\square$

*Proof (of Proposition 4)* Item 4 is obvious from the definition of $C(v)$. To show item 1, rewrite the function $C(v)$ as

$$\begin{aligned} C(v) &= \frac{\gamma}{r} \left[ \frac{\delta A(v_m) + r A(v)}{A(v_m) - A(v_0)} \right] \\ &= \frac{\gamma}{r} \left[ \delta + \frac{\delta A(v_0) + r A(v)}{A(v_m) - A(v_0)} \right] \end{aligned}$$

which is clearly increasing in $v_m$ since $A(\cdot)$ is decreasing. The terms $\omega$ and $\eta$ only appear in the definition of $v_0 = \omega/\eta$. Since $C(\cdot)$ is clearly decreasing in $v_0$, it is also decreasing in $\omega$ and increasing in $\eta$. This concludes the proof. $\square$

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